UNIVERSITÀ DI PISA

DIPARTIMENTO DI INGEGNERIA DELL'INFORMAZIONE

Dottorato di Ricerca in Ingegneria dell'Informazione

Doctoral Course

"Advanced Algebraic Methods for Robotics and AI"

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Short Abstract:

Several problems in advanced engineering rely on the manipulation of vectors or matrices that are functions of other vectors or matrices, and their derivatives and differentials. Examples arise in various fields spanning Analytical Mechanics, Robot Control, Deep Neural Networks, Optimization and Computer Vision.

Although the such objects are studied since more than a century, modern theory of matrix algebra and matrix calculus introduced, based the Kronecker product and matrix vectorization, simple yet unambiguous, therefore powerful and flexible, definitions of matrix derivatives and matrix differentials. Those tools have been successfully used to obtain novel theoretical results in the fields of Probability, Psychometrics and Econometrics, by enabling the formal manipulation of complex expressions.

This course will introduce such mathematical instruments in detail and present their application to several relevant problems encountered in the aforementioned engineering fields, to streamline their description and yield more elegant and compact derivations, that will hopefully enable advanced student of those fields to achieve novel and significant results.

Course Contents in brief:

- Matrix vectorization, Kronecker product, Trace and their properties
- Definition of Matirx Derivatives and Matrix Differentials and some Notable Results
- Applications to Analytical Mechanics: Rotation differentials, Robots Jacobian and Stiffness
- Applications to Robot Control: Lagrange Dynamics, Regressor Form of Robot Dynamics
- Applications to AI: description of a Deep Neural Network, Computer Vision.

Total # of hours of lecture: 20 h

References:

[1] Magnus, Jan R., and Heinz Neudecker. *Matrix differential calculus with applications in statistics and econometrics*. John Wiley & Sons, 2019.

[2] Graham, Alexander. *Kronecker products and matrix calculus with applications*. Courier Dover Publications, 2018.

CV of the Teacher

Giorgio Grioli received his PhD in Robotics, Automation and Bio-Engineering from the University of Pisa in 2011, with a thesis on identification for control of variable impedance actuators. He is the author of more than 130 scientific papers, published in scientific journals and international conference proceedings, in the fields of soft robotic actuation, robotic hand design, haptics, human-machine interaction. Co-inventor of numerous robotic devices, helped found two spin-off companies. He has served as Associate Editor for the ICRA and ICORR conferences (since 2015) and as Editor for MDPI – Actuators, Cambridge – Robotics, and Springer IJRR journals. Over the years he supervised the development of over 40 master's theses in the Automation Engineering and Mechanical Engineering courses and several bachelor's theses and student projects for the Robotics course. Member of the Information Engineering Doctoral Board of the University of Pisa, he succesfully supervised 6 students and is currently supervising 5 more. He also supervised a PhD student in Smart Industries and is supervising two PhD students on the PhD course national interest in Robotics and Intelligent Machines DRIM. Since September 2023 he has been appointed Senior Researcher at the University of Pisa, where he teaches "robot control" for the "robotic and automation engineering" masters' degree course and "automatic controls" for the "vehicle engineering" masters' degree course.

Final Exam:
Student project defined in accordance with the teacher.

Room and Schedule

Room: *Aula Riunioni del Dipartimento di Ingegneria dell'Informazione, Via G. Caruso 16, Pisa – Ground Floor*

Schedule:

June 23, 2025, 9:00-13.00:

1. Matrix vectorization, Kronecker product, Trace and their properties (2h)
2. Magnus definition of derivatives and their application to matrix calculus (1h)
3. Series expansion of a vector field (1h)

June 24, 2025, 9:00-13:00:

4. Derivative of matrix inverse, matrix determinant and matrix pseudo-inverse (2h)
5. Integration of angular speeds into rotation matrices, definition of angular error (1h)
6. Jacobians: algebraic, geometric, and geometric from algebraic (1h)

June 25, 2025, 9:00-13:00:

7. Jacobians: algebraic, geometric, and geometric from algebraic (1h)
8. Stiffness of a kinematic chain (2h)

9. Second Order and Higher Order Differential Kinematics (1h)

June 26, 2025, 9:00-13:00:

10. Lagrangian dynamics with kron and vec (2h)
11. Robot Kinematics and Dynamics in Regressor Form: either and both (2h)

June 27, 2025, 9:00-13:00:

12. Deep Neural Networks and Back-propagation (2h)
13. Derivative of ReLU, MAX, softMAX, and softReLU (1h)
14. Exercise: Neuromorphic Adaptive control with arbitrary gain matrix (1h)